

Bulk viscosity-driven suppression of shear viscosity effects on the flow harmonics at RHIC

Phys. Rev. C**88** (2013) 044916

Phys. Rev. C**90** (2014) 034907

arXiv:1411.2574

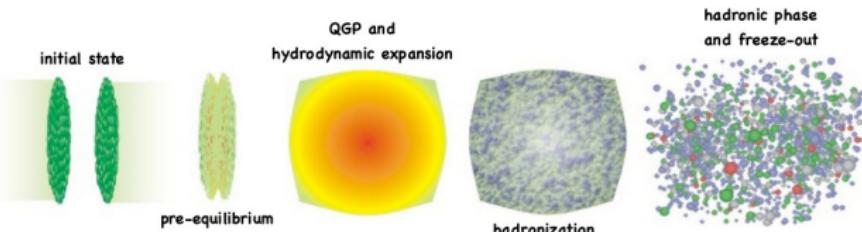
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USP-McGill-Santiago-Columbia Collaboration

BNL-RIKEN Lunch Seminar Nov. 20, 2014

Outline

- 1 Heavy-Ion Collisions
- 2 Effects of Viscosity
- 3 Viscous Rel. Hydro Event by Event
- 4 Results
- 5 Conclusions

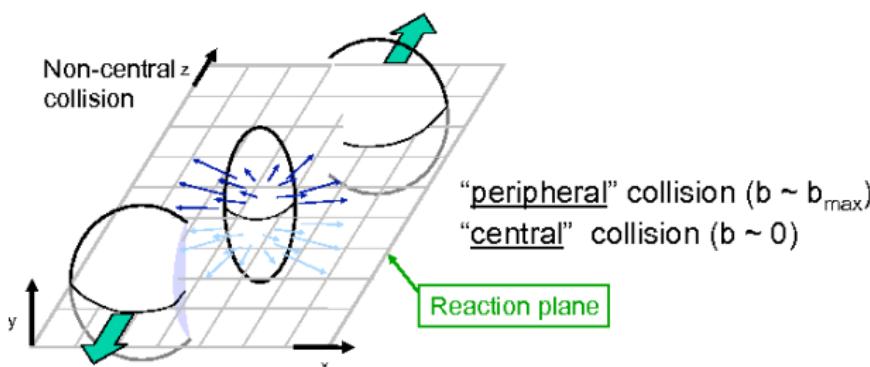
Evolution of a Heavy-Ion Collision



Heavy ion collisions are modeled through

- Initial Condition: Pre-equilibrium state using gluon saturation models/Glauber-like models
- Viscous hydrodynamical evolution/Lattice Equation of State
- Hadronization mechanism: Cooper Frye including viscous corrections
- Hadronic afterburner

Heavy-Ion Collision



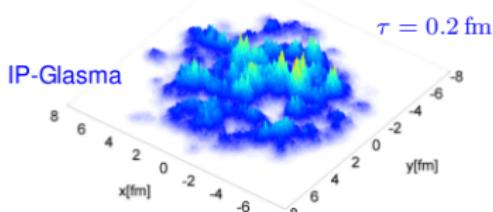
- Central Collisions 0 – 5% (small impact parameter $b < 3$ fm) have small eccentricity
 - Non-central Collisions 20 – 30% (mid-sized impact parameter) have larger eccentricity
 - Peripheral Collisions 60 – 70% (large impact parameter $b \sim b_{max}$) small eccentricity- not enough matter

Types of Initial Conditions

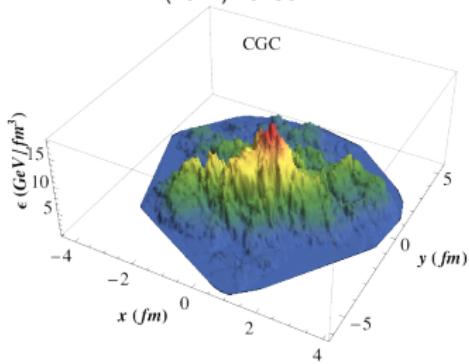
- Glauber model: optical geometric model determining wounded nucleons based on the inelastic cross section. Only sees the effect of the nucleons, no initial flow: **Coarse Structure**
 - Color-Glass Condensate-like models: Considers effect of the wounded nucleons as in Glauber and also their color charges within, no initial flow: **Finer Structure**
 - IP-Glasma model: New saturation-based model using classical Yang-Mills evolution of early-time gluon fields, including additional fluctuations in the particle production, non-zero initial flow: **Finest Structure**
 - UrQMD, **NEXUS**, BAMPS: hadronic or partonic cascades can also provide initial conditions

Types of Initial Conditions

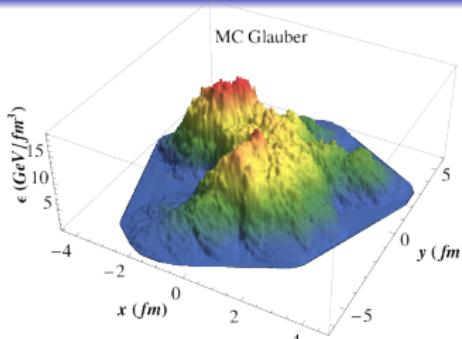
Energy Density profile



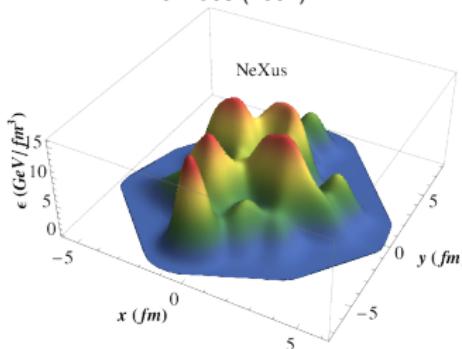
Schenke, Tribedy, Venugopalan, Phys. Rev. Lett. 108
(2012) 252301



Drescher,Nara, Phys. Rev. C 75, 034905 (2007); ,76,
041903 (2007).



Drescher,Nara, Phys. Rev. C 75, 034905 (2007); ,76, 041903 (2007).



Drescher, Hladik, Ostapchenko, Pierog, Werner, Phys. Rept.
350, 93 (2001)

Initial Conditions effects on Collective Flow

The distribution of particles can be written as a Fourier series (event plane method)

$$E \frac{d^3N}{d^3p} = \frac{1}{2\pi} \frac{d^2N}{p_T dp_T dy} \left[1 + \sum_n 2v_n \cos [n(\phi - \psi_n)] \right]$$

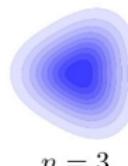
- Flow Harmonics at mid-rapidity

$$v_n(p_T) = \frac{\int_0^{2\pi} d\phi \frac{dN}{p_T dp_T d\phi} \cos[n(\phi - \Psi_n)]}{\int_0^{2\pi} d\phi \frac{dN}{p_T dp_T d\phi}}$$

where $\Psi_n = \frac{1}{n} \arctan \frac{\langle \sin[(n\phi)] \rangle}{\langle \cos[(n\phi)] \rangle}$



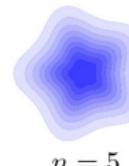
$$n = 2$$



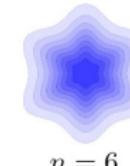
$$n = 3$$



$$n = 4$$



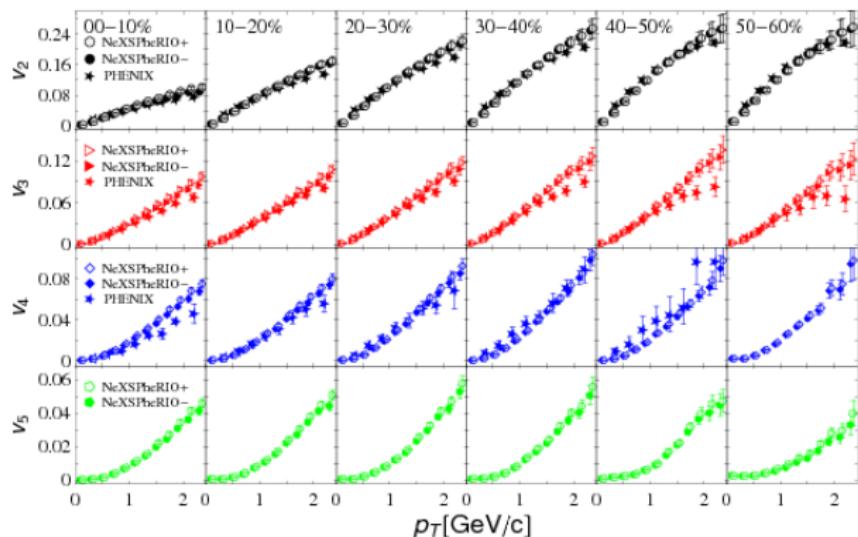
n = 5



$$n = 6$$

Initial Conditions effects on Collective Flow

- Event-by-event NeXus initial conditions and 3+1 ideal relativistic hydrodynamics fit the flow harmonics well



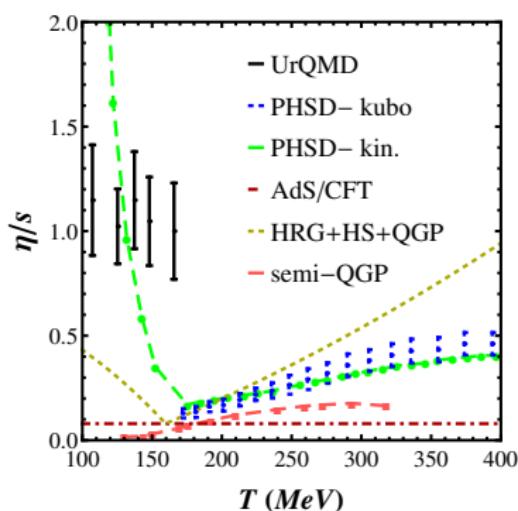
Gardim, Grassi, Luzum, Ollitrault, Phys.Rev.Lett. 109 (2012) 202302

Shear Viscosity in Heavy-Ion Collisions

- Resistance against the deformation of a fluid

$$\Pi_{\text{Navier-Stokes}}^{\mu\nu} \sim \eta \partial^{\langle \mu} u^{\nu \rangle}$$

Minimum at T_c :



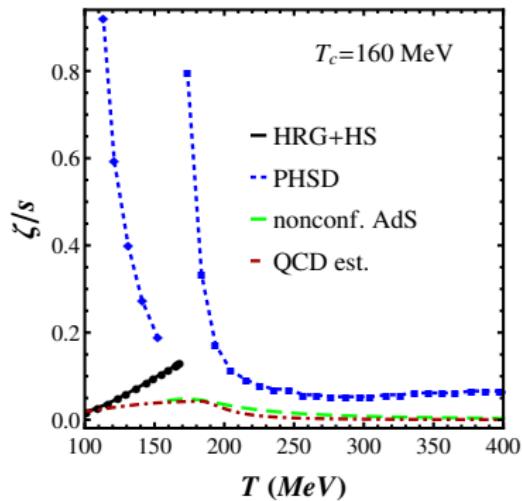
- HRG+HS+QGP(JNH et al PRL103(2009)172302, Niemi et al PRL106(2011)212302)
- PHSD (PRC87(2013)064903)
- AdS/CFT -KSS limit (Kovtun,Son,Stainnets PRL94(2005)111601)
- UrQMD (Demir, Bass PRL(2009)102)
- semi-QGP- $\kappa = 32$ (Hidaka,Pisarski PRD81(2010)076002)
- Also, Csernai,Kapusta,McLerran PRL 97, 152303 (2006) (minimum suggested-not shown)

Bulk Viscosity in Heavy-Ion Collisions

- Resistance against the radial expansion or compression of a fluid $\Pi_{\text{Navier-Stokes}} \sim -\zeta(\partial_\mu u^\mu)$
- Evolution with a non-zero ζ/s slows down the expansion of the fluid.
- Previous assumption: ζ/s is negligible in hydrodynamics studies of heavy-ion collisions

Bulk Viscosity in Heavy-Ion Collisions

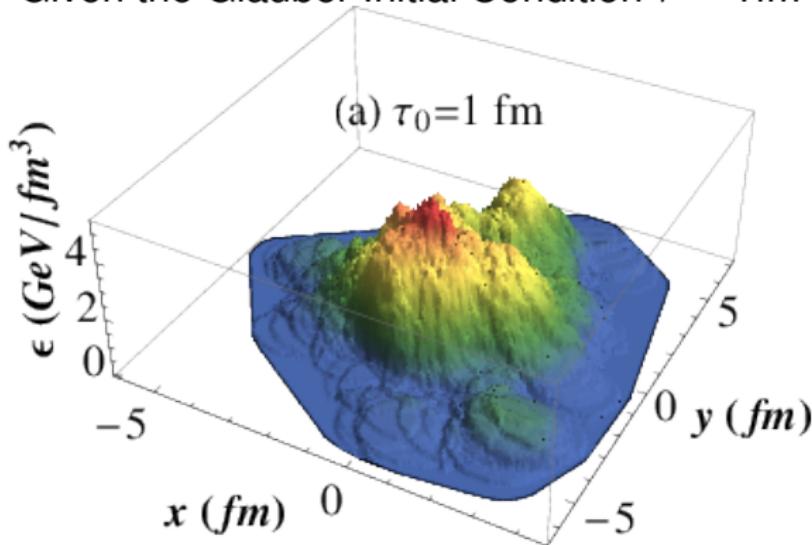
- Resistance against the radial expansion or compression of a fluid $\Pi_{\text{Navier-Stokes}} \sim -\zeta(\partial_\mu u^\mu)$
- Peak at T_c ?



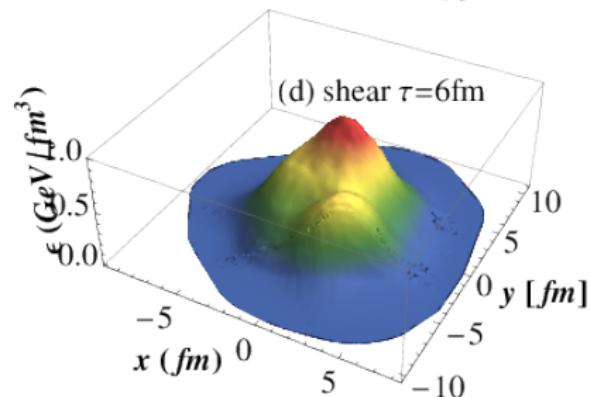
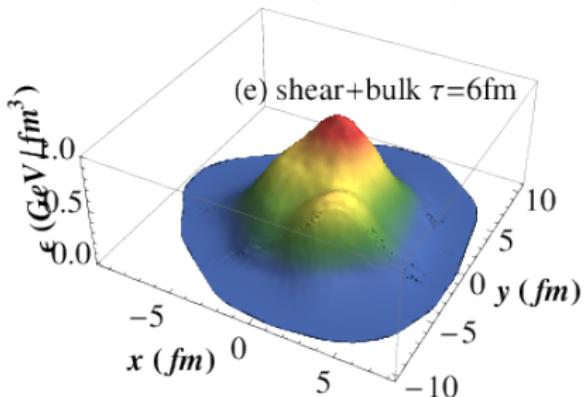
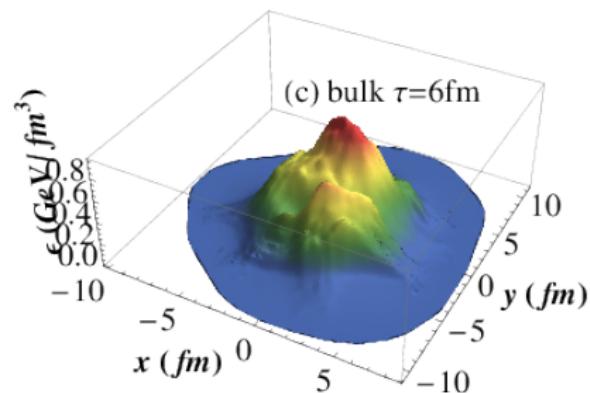
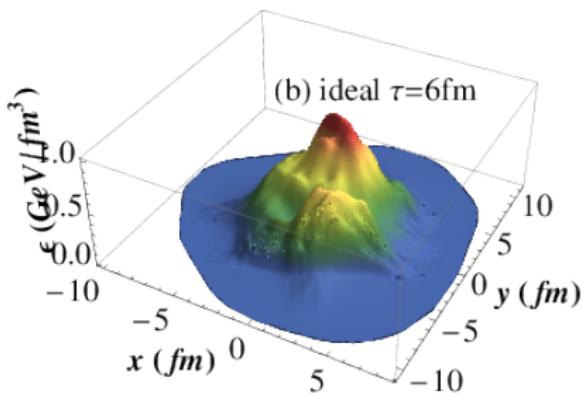
From:
HRG+HS(Kadam and Mishra
arXiv:1408.6329)
PHSD (PRC 87, 064903 (2013))
non-conformal holographic model
(Finazzo, Rougemont, Noronha - to
appear shortly)
pQCD (Arnold, Dogan, Moore
Phys.Rev. D74 (2006) 085021)
Peak also seen in:
JNH, PRL 103 (2009) 172302,
Kharzeev JHEP 0809 (2008) 093

Viscosity in Heavy-Ion Collisions

Given the Glauber Initial Condition $\tau = 1 \text{ fm}$



Viscosity in Heavy-Ion Collisions



v-USPhydro - viscous Ultrarelativistic Smoothed Particle hydrodynamics

JNH et al, PRC90(2014)034907, PRC88(2013)044916

Motivation

Write a modular event-by-event 2+1 hydrodynamical code that runs ideal & viscous hydro with nonzero ζ/s and η/s

- Initial conditions can easily be implemented from other sources.
- The equations of motion are solved using Smoothed Particle Hydrodynamics (SPH)- quick computational time and avoids certain problems (numerical viscosity, grid size issues etc)
- Coupled to UrQMD but results here are only without decays.

Equations of Motion

Conservation of Energy and Momentum

$$\partial_\mu T^{\mu\nu} = 0 \quad (1)$$

The energy-moment tensor contains a bulk dissipative term Π and the shear stress tensor $\pi^{\mu\nu}$ is

$$T^{\mu\nu} = \varepsilon u^\nu u^\nu - (p + \Pi) \Delta^{\mu\nu} + \pi^{\mu\nu} \quad (2)$$

Coordinate System: $x^\mu = (\tau, x, y, \eta)$ where $\tau = \sqrt{t^2 - z^2}$ and
 $\eta = 0.5 \ln \left(\frac{t+z}{t-z} \right)$

Smoothed Particle Hydrodynamics (SPH) Overview

Motivation

SPH discretizes the fluid into a number of SPH particles whose trajectories (\mathbf{r} and \mathbf{u}) you observe over time

Imagine you want to observe the motion of a lake

- SPH (Lagrangian)- you are in a boat on the lake and move over the coarse of time watching your trajectory.
- Grid (Euler)- you are seated at a dock and observe the rise and fall of water at a set spot 2m away from you.

Equations of Motion

- SPH conserves reference density current: $J^\mu = \sigma u^\mu$ where σ is the local density of a fluid element in its rest frame
- Density obeys $\partial_\mu(\tau\sigma u^\mu) = 0$ in hyperbolic coordinates (in Cartesian $D\sigma + \sigma\theta = 0$) where $D = u^\mu\partial_\mu$ and $\theta = \tau^{-1}\partial_\mu(\tau u^\mu)$
- We use this set of IS equations, which provides the simplest equations for viscous hydrodynamics.

$$\begin{aligned}\tau_\Pi(D\Pi + \Pi\theta) + \Pi + \zeta\theta &= 0, \\ \tau_\pi \left(\Delta_{\mu\nu\alpha\beta} D\pi^{\alpha\beta} + \frac{4}{3}\pi_{\mu\nu}\theta \right) + \pi_{\mu\nu} &= 2\eta\sigma_{\mu\nu}\end{aligned}$$

PRC75(2007) 034909

- There are four transport coefficients: η/s , ζ/s , τ_π , and τ_Π

Description of Shear and Bulk Viscosity

$$\begin{aligned}\frac{\eta}{s}(T > T_{tr}) &= -0.289 + 0.288 \left(\frac{T}{T_{tr}} \right) + 0.0818 \left(\frac{T}{T_{tr}} \right)^2 \\ \frac{\eta}{s}(T < T_{tr}) &= 0.681 - 0.0594 \left(\frac{T}{T_{tr}} \right) - 0.544 \left(\frac{T}{T_{tr}} \right)^2\end{aligned}$$

JNH

PBL 103(2009)172302 PBC 86(2012)014909&PBL 106(2011)212302

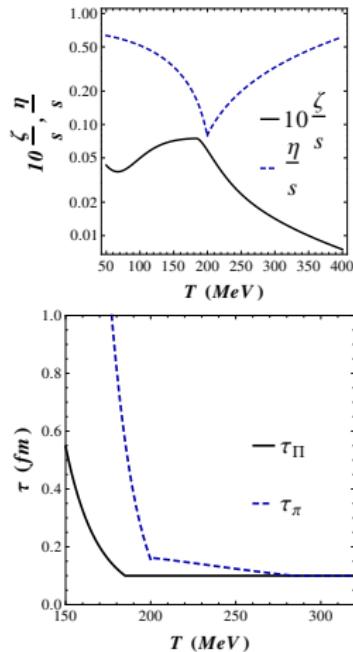
$$\tau_\pi = 5\eta/(\varepsilon + p)$$

PRL105, 162501 (2010)

$$\left(\frac{\zeta}{s}\right) = 0.5 \frac{\eta}{s} \left(\frac{1}{3} - c_s^2\right), \quad \pi_1 = 9 \frac{\zeta}{\epsilon - 3p}$$

BuchelPLB663(2008)286

Huang,Kodama,Koide,RischkePRC83(2011)024906



Parameters

- Isothermal freeze-out temperature: $T_{FO} = 150$ MeV
- Initial time to start hydrodynamic simulation: $t_0 = 1$ fm
- Lattice-based equation of state from Huovinen&Petreczky,
NPA837, 26(2010)
Currently testing HotQCD PRD90(2014)9,094503
- SPH scale $h = 0.3$ fm
- Energy conservation for event-by-event Glauber initial conditions: Ideal case $\sim 0.001\%$, Viscous case $\sim 0.1\%$

Shear effect on bulk (hydro only)

Percent change of mean and variance in the presence of shear+bulk vs. bulk only

$$(\Pi)_{ev} = 100 \frac{(\Pi_{sb})_{ev} - (\Pi_b)_{ev}}{(\Pi_b)_{ev}}$$

$$(\sigma_{\Pi}^2)_{ev} = 100 \frac{\left(\sigma_{\Pi_{sb}}^2\right)_{ev} - \left(\sigma_{\Pi_b}^2\right)_{ev}}{\left(\sigma_{\Pi_b}^2\right)_{ev}}$$

	$\langle \Pi \rangle$	σ_{Π}^2	$\langle \Pi \rangle_{early}$	$(\sigma_{\Pi}^2)_{early}$	$\langle \Pi \rangle_{late}$	$(\sigma_{\Pi}^2)_{late}$
0-10%	1.79%	8.59%	1.14%	-59.72%	2.03%	20.50%
10-20%	2.48%	8.95%	2.89%	-52.37%	2.19%	20.59%
20-30%	2.87%	8.96%	4.07%	-40.70%	2.02%	20.66%
30-40%	3.49%	9.15%	3.47%	-36.96%	2.15%	19.97%
40-50%	4.14%	9.11%	3.52%	-37.23%	2.00%	20.86%
50-60%	4.98%	9.23%	6.27%	-22.55%	2.28%	19.73%

TABLE II. Percentage change of the mean values of the bulk pressure Π and its corresponding variance σ_Π^2 averaged over all events for different centrality classes due to the presence of shear viscosity. $\langle \Pi \rangle$ and σ_Π^2 takes into account the parts of the fluid that have frozen out throughout the whole time evolution, $\langle \Pi \rangle_{early}$ and $(\sigma_\Pi^2)_{early}$ are computed using only the parts of the fluid that have frozen out between $\tau_0 = 1$ fm and $\tau = 2$ fm, $\langle \Pi \rangle_{late}$ and $(\sigma_\Pi^2)_{late}$ are computed using only the parts of the fluid that have frozen out in the last fm of the time evolution.

- Shear increases the variation in bulk (at late times)

Bulk effect on shear (hydro only)

Centrality	$\langle \pi^{00} \rangle$	$\sigma_{\pi^{00}}^2$	$\langle \pi^{12} \rangle$	$\sigma_{\pi^{12}}^2$
0-10%	-17.61%	-19.09%	-2.87%	-8.50%
10-20%	-17.77%	-18.53%	-2.25%	-8.45%
20-30%	-19.22%	-18.56%	-3.48%	-8.44%
30-40%	-22.98%	-18.53%	-3.26%	-8.35%
40-50%	-38.11%	-19.37%	-2.81%	-8.01%
50-60%	-44.63%	-19.61%	-5.05%	-7.68%

TABLE III. The percentage change in the mean values and variance of the π^{00} and π^{12} components of the shear stress tensor $\pi^{\mu\nu}$ averaged over all events and all SPH particles due to the inclusion of bulk viscosity in the time evolution. These quantities are computed taking into account the parts of the fluid that have frozen out throughout the whole time evolution.

Centrality	$\langle \pi^{00} \rangle_{early}$	$(\sigma_{\pi^{00}}^2)_{early}$	$\langle \pi^{12} \rangle_{early}$	$(\sigma_{\pi^{12}}^2)_{early}$
0-10%	-6.66%	-12.79%	-5.94%	-10.66%
10-20%	-5.32%	-11.31%	-4.87%	-9.46%
20-30%	-6.07%	-12.72%	-4.81%	-9.15%
30-40%	-7.01%	-14.08%	-4.80%	-9.19%
40-50%	-4.75%	-9.00%	-4.75%	-8.99%
50-60%	-6.83%	-15.02%	-4.63%	-8.76%

TABLE IV. The percentage change in the mean values and variance of the π^{00} and π^{12} components of the shear stress tensor $\pi^{\mu\nu}$ averaged over all events and all SPH particles due to the inclusion of bulk viscosity in the time evolution. These quantities are computed taking into account only the parts of the fluid that have already frozen for early times (between $\tau = \tau_0$ and $\tau = 2$ fm).

Centrality	$\langle \pi^{00} \rangle_{late}$	$(\sigma_{\pi^{00}}^2)_{late}$	$\langle \pi^{12} \rangle_{late}$	$(\sigma_{\pi^{12}}^2)_{late}$
0-10%	-17.68%	-29.13%	-5.94%	-10.80%
10-20%	-15.98%	-29.09%	-4.80%	-9.38%
20-30%	-15.45%	-28.56%	-4.77%	-9.06%
30-40%	-14.97%	-28.28%	-4.88%	-9.34%
40-50%	-13.83%	-27.91%	-4.80%	-9.20%
50-60%	-12.75%	-26.18%	-4.50%	-8.51%

TABLE V. The percentage change in the mean values and variance of the π^{00} and π^{12} components of the shear stress tensor $\pi^{\mu\nu}$ averaged over all events and all SPH particles due to the inclusion of bulk viscosity in the time evolution. These quantities are computed taking into account only the parts of the fluid that have frozen during the last fm of the time evolution.

- Bulk suppresses the $\pi^{\mu\nu}$ and has the largest affect at late times.

Cooper-Frye Freeze-out

Overview

$$\left(E_p \frac{dN}{d^3p}\right)_i = g_i \int_{\Sigma} d\Sigma_{\mu} p^{\mu} f_i$$

Particle distribution function:

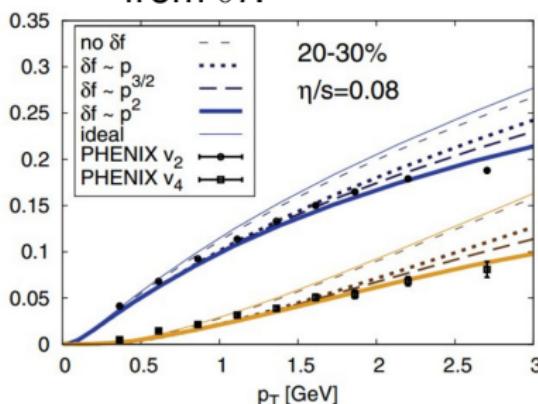
$$f_{\mathbf{k}}^{(i)} = f_{0\mathbf{k}}^{(i)} + \delta f_{\mathbf{k}}^{(i)}$$

$$f_{0\mathbf{k}}^{(i)} = (\exp[E_i/T] + a_i)^{-1}$$

Fermions: $a_i = 1$, Bosons: $a_i = -1$

Boltzmann gas: $a_i = 0$

Note that majority of viscous effects come from δf .



Schenke, Jeon, Gale, PRC85(2012)024901

Cooper-Frye Freeze-out

Derivation of $\delta f_{\mathbf{k}}^{(i)}$ 1/2: Denicol et al, PRD85(2012)114047

Particle distribution function computed using a version of Grad's 14 moment approximation for the Boltzmann equation:

- Factorize $\delta f_{\mathbf{k}}^{(i)}$: $\delta f_{\mathbf{k}}^{(i)} = f_{0\mathbf{k}}^{(i)} \tilde{f}_{0\mathbf{k}}^{(i)} \phi_{\mathbf{k}}^{(i)}$ where $\tilde{f}_{0\mathbf{k}}^{(i)} = 1 + af_{0\mathbf{k}}^{(i)}$
- Determine $\phi_{\mathbf{k}}^{(i)}$, out of equilibrium contribution, by establishing a basis of

Irreducible Tensors: $k_i^{\langle\mu\rangle}, k_i^{\langle\mu} k_j^{\nu\rangle}, k_i^{\langle\mu} k_j^{\nu} k_i^{\lambda\rangle}, \dots,$

Orthonormal Polynomials: $P_{i\mathbf{k}}^{(n\ell)} = \sum_{r=0}^n a_{nr}^{(\ell)i} (u_\mu k_i^\mu)^r,$

- Then, $f_{\mathbf{k}}^{(i)} = f_{0\mathbf{k}}^{(i)} + f_{0\mathbf{k}}^{(i)} \tilde{f}_{0\mathbf{k}}^{(i)} \sum_{\ell=0}^{\infty} \sum_{n=0}^{\infty} \mathcal{H}_{i\mathbf{k}}^{(n\ell)} \rho_{i,n}^{\mu_1 \dots \mu_\ell} k_{i,\mu_1} \dots k_{i,\mu_\ell}$
where $\mathcal{H}_{i\mathbf{p}}^{(n\ell)} \equiv [N_i^{(\ell)} / \ell!] \sum_{m=n}^{\infty} a_{mn}^{(\ell)i} P_{i\mathbf{k}}^{(m\ell)} (u_\mu k_i^\mu)$

Cooper-Frye Freeze-out

Derivation of $\delta f_{\mathbf{k}}^{(i)}$ 2/2: Denicol et al, PRD85(2012)114047

Truncating in momentum space up to the 2nd order and using the orthogonality relations from the basis:

$$f_{\mathbf{k}}^{(i)} = f_{0\mathbf{k}}^{(i)} + \delta f_{\mathbf{k}}^{(i) \text{ Bulk}} + \delta f_{\mathbf{k}}^{(i) \text{ Shear}},$$

$$\delta f_{\mathbf{k}}^{(i) \text{ Shear}} = \frac{f_{0\mathbf{k}}^{(i)}}{2(\varepsilon_i + P_i) T^2} \frac{\eta_i}{\eta} \pi^{\mu\nu} k_{i,\mu} k_{i,\nu},$$

$$\delta f_{\mathbf{k}}^{(i) \text{ Bulk}} = f_{0\mathbf{k}}^{(i)} \Pi \left[B_0^{(i)} + D_0^{(i)} u \cdot k_i + E_0^{(i)} (u \cdot k_i)^2 \right]$$

- $E_{0,i}$, $D_{0,i}$, $B_{0,i}$: functions of mass m_i and T - determined through basis

Cooper-Frye Freezeout

Major Assumptions

- We assume Navier-Stokes scaling to relate the moments $\rho_{i,0}$, $\rho_{i,2}$, $\rho_{i,0}^{\mu\nu}$ to Π and $\pi^{\mu\nu}$ - neglect effects from τ_Π .

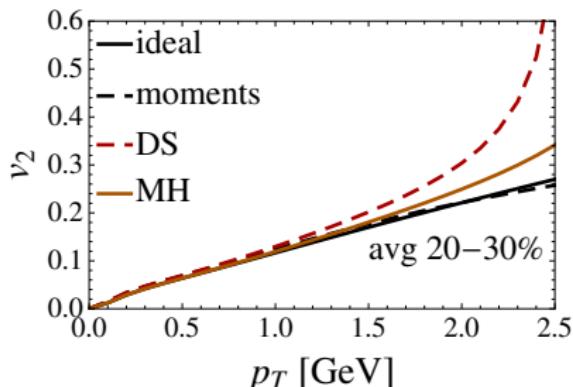
$$\Pi = -\zeta \partial_\mu u^\mu, \rho_{i,m} = -\alpha_{i,m} \partial_\mu u^\mu \implies \rho_{i,m} = \frac{\alpha_{i,m}}{\zeta} \Pi,$$

$$\pi_i^{\mu\nu} = 2\eta_i \partial^{\langle\mu} u^{\nu\rangle}, \pi_i^{\mu\nu} = 2\eta \partial^{\langle\mu} u^{\nu\rangle} \implies \pi_i^{\mu\nu} = \frac{\eta_i}{\eta} \pi^{\mu\nu}.$$

- All hadrons have the same cross-section of 30 mb
- Only hadrons up to a mass of $M = 1.2$ GeV are considered (every additional hadron increases the matrix rank needed for the calculation of transport coefficients, which becomes very costly)
- Freeze-out temperature $T_{FO} = 150$ MeV.

Dependence on δf - bulk only

JNH PRC88(2013)044916



$$\delta f_{\mathbf{k}}^{(\pi)} = f_{0\mathbf{k}}^{\pi} \Pi * \left[B_0^{(\pi)} + D_0^{(\pi)} u \cdot k_{\pi} + E_0^{(\pi)} (u \cdot k_{\pi})^2 \right]$$

Averaged Glauber

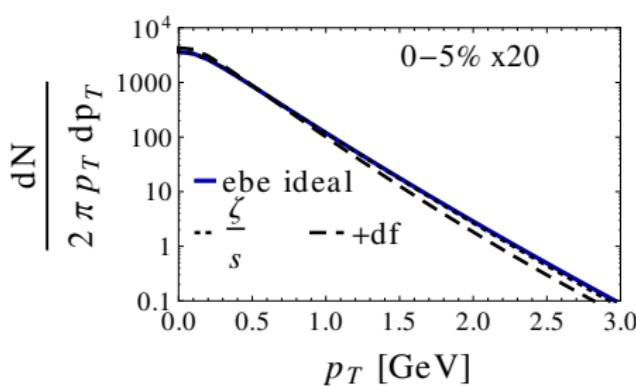
	E_0 [fm^4]	D_0 [$\frac{fm^4}{GeV}$]	B_0 [$\frac{fm^4}{GeV^2}$]	
mo	-65.85	171.27	-63.05	PRC88(2013)044916
DS	-71.96	121.50	0	PRC85(2012)044909
MH	-0.69	-38.96	49.69	PRC80(2009)054906

π^+ Spectrum (Direct π^+ 's Only)

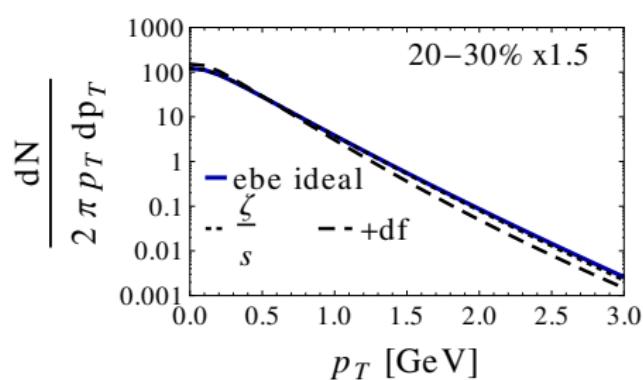
JNH PRC88(2013)044916

At $T = 150$ MeV about 41% of pions are direct pions. For most central collisions there are about 300 π^+ 's, so 123 direct π^+ 's.

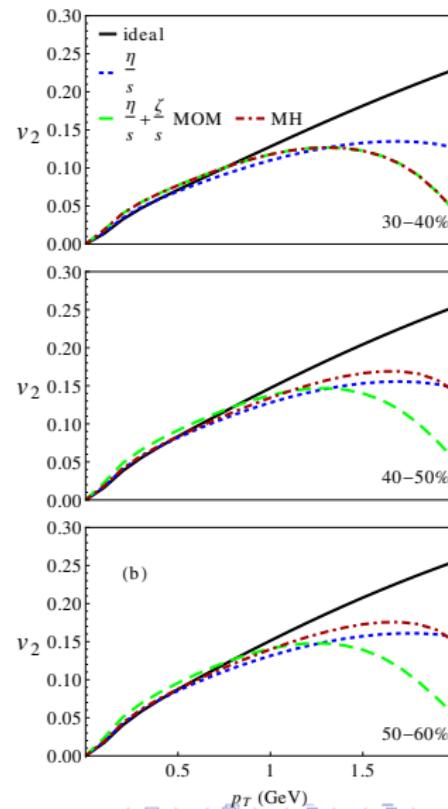
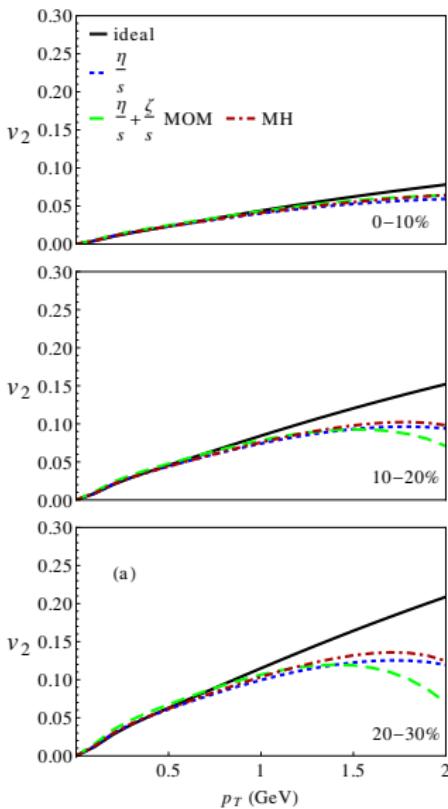
$$\pi^+ \approx 123$$



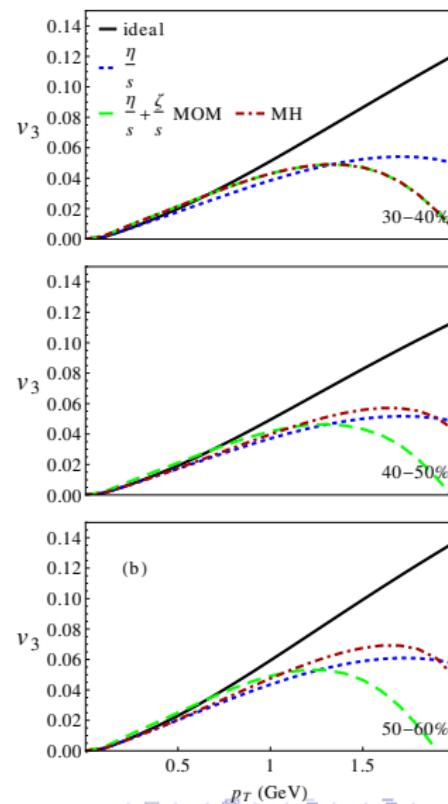
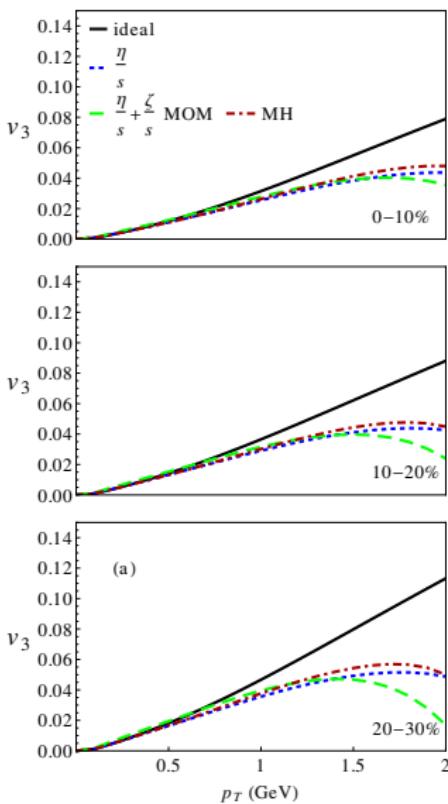
$$\pi^+ \approx 54$$



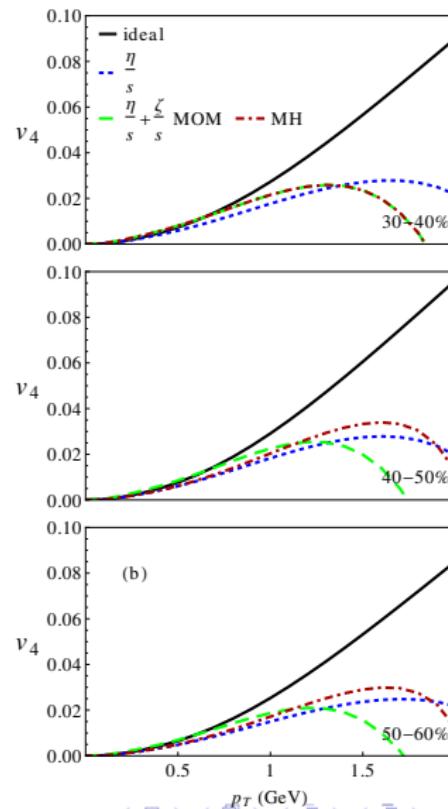
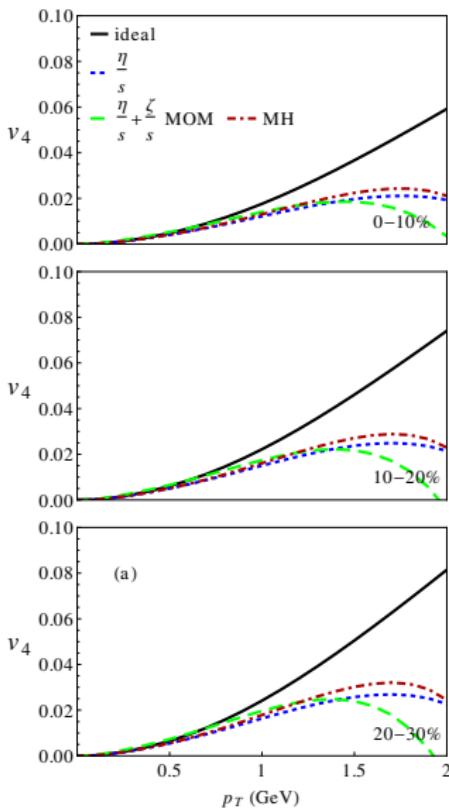
Event-by-Event v_2
JNH PRC90(2014)034907



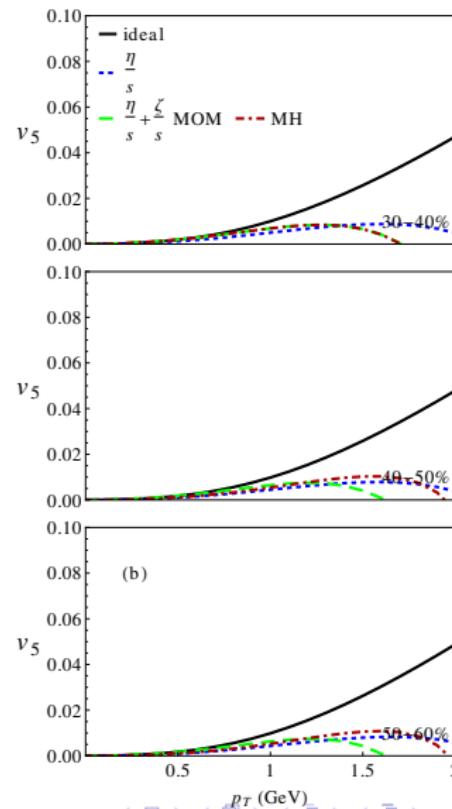
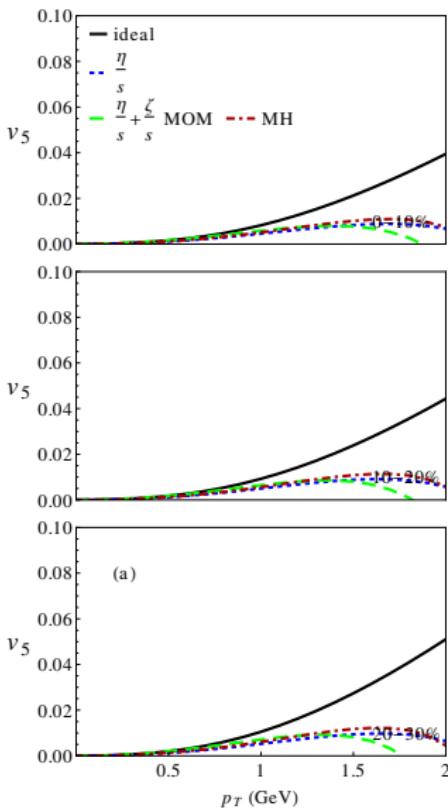
Event-by-Event v_3
JNH PRC90(2014)034907



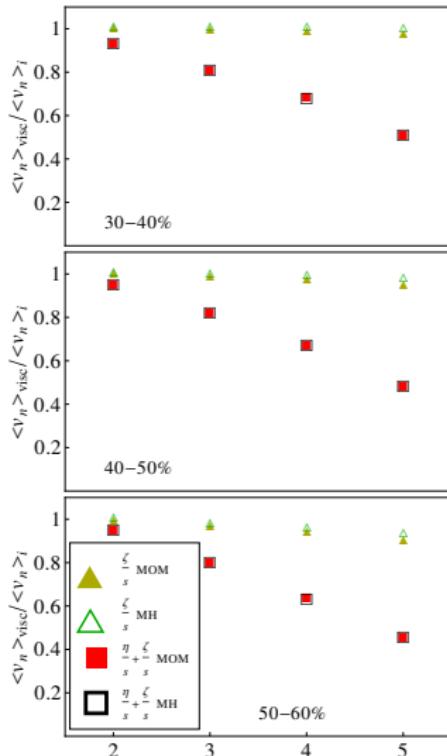
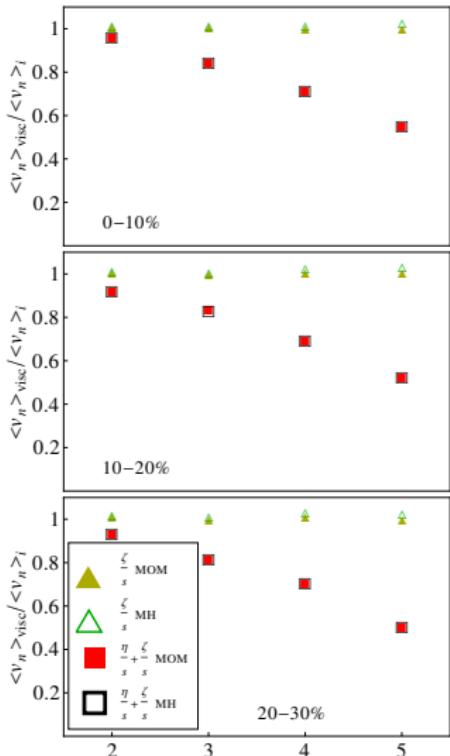
Event-by-Event v4
JNH PRC90(2014)034907



Event-by-Event v_5
JNH PRC90(2014)034907

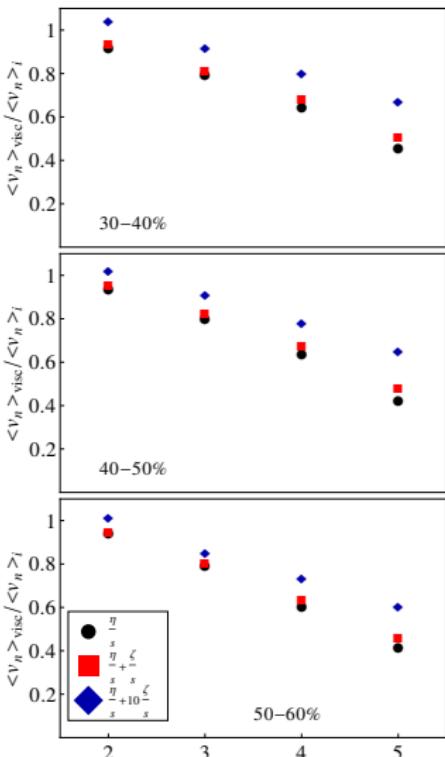
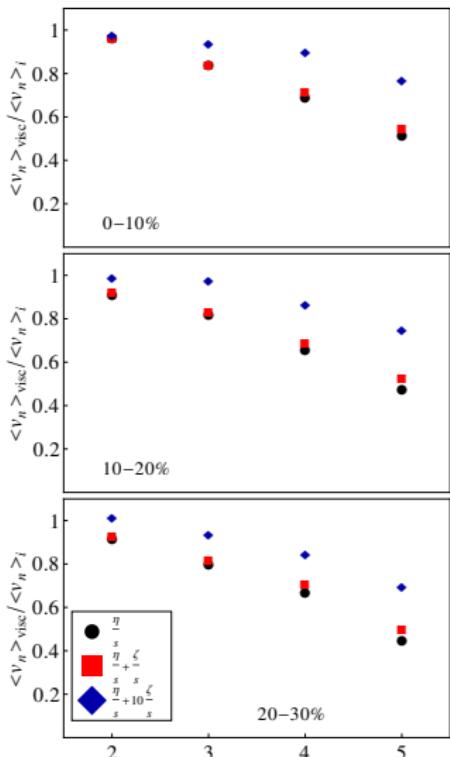


Integrated ν_n 's - Comparing δf
JNH PRC90(2014)034907



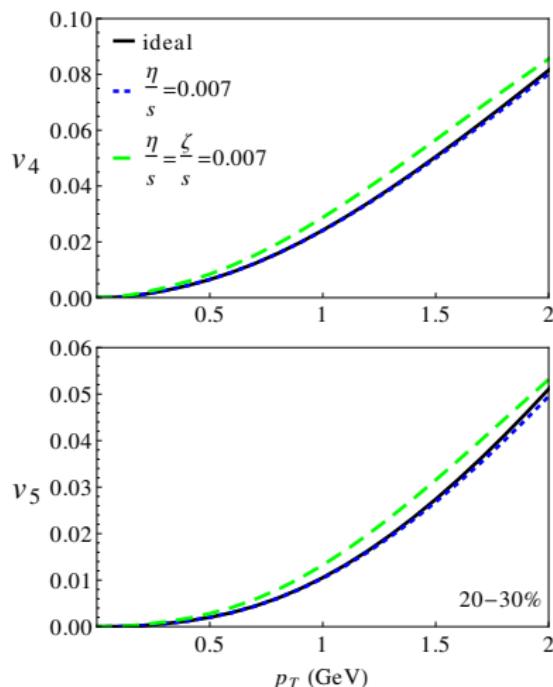
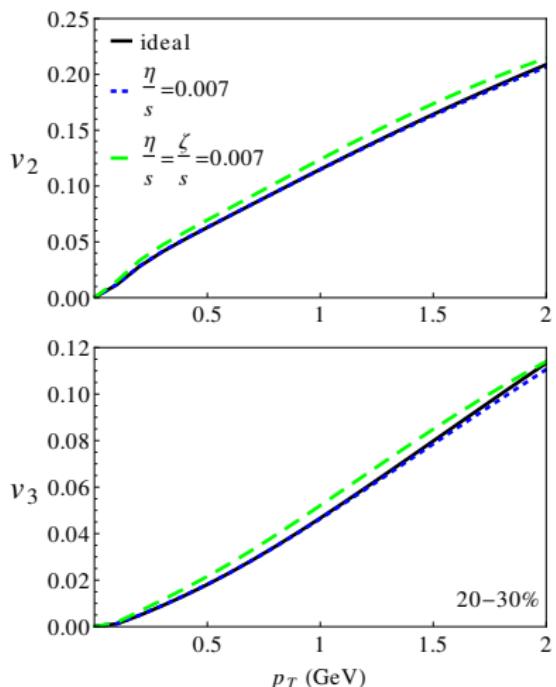
Integrated v_n 's - Comparing ζ/s

JNH PRC90(2014)034907



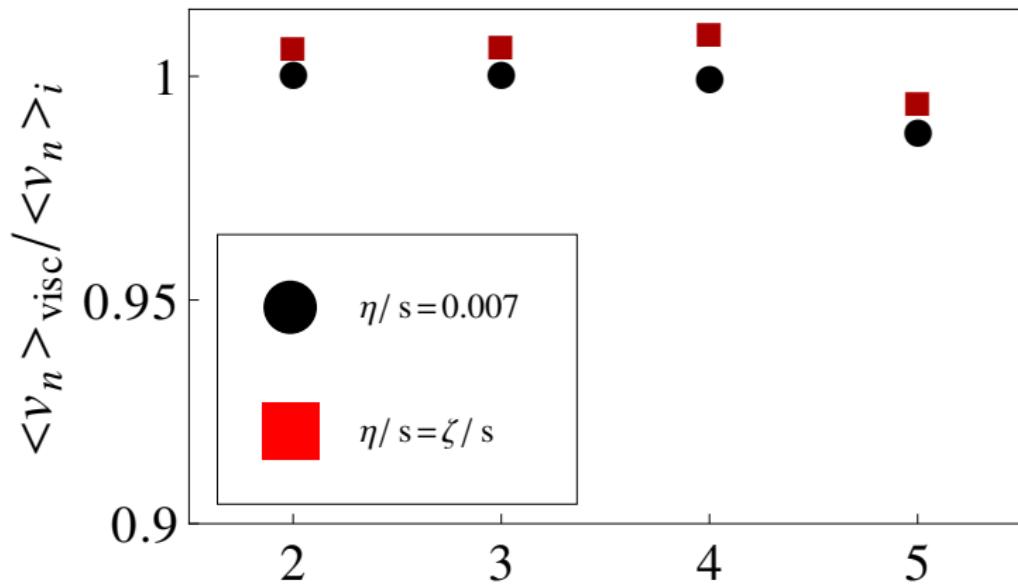
$\zeta/s = \eta/s$ p_T dependent

JNH PRC90(2014)034907



$\zeta/s = \eta/s$ integrated

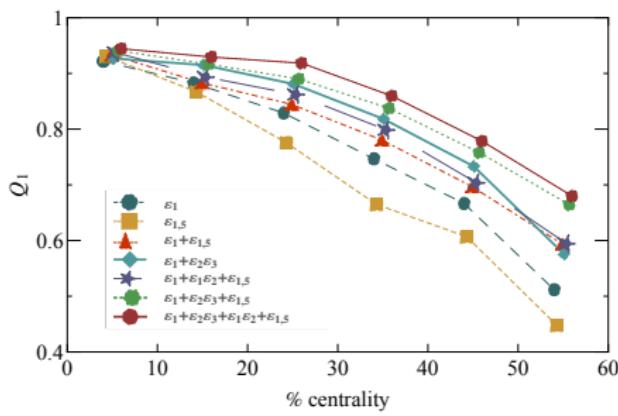
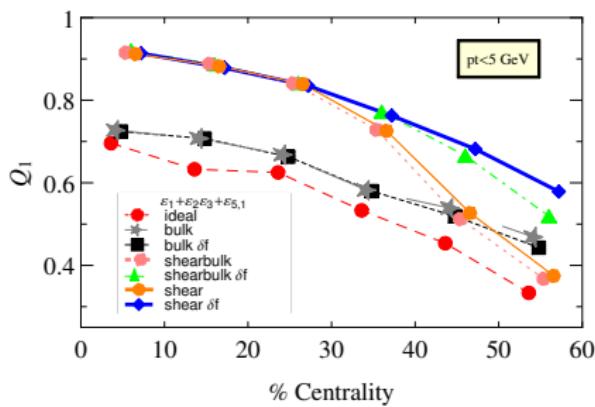
JNH PRC90(2014)034907



v_1 from $\varepsilon_1 + \varepsilon_2\varepsilon_3 + \varepsilon_1\varepsilon_5$

JNH arXiv:1411.2574

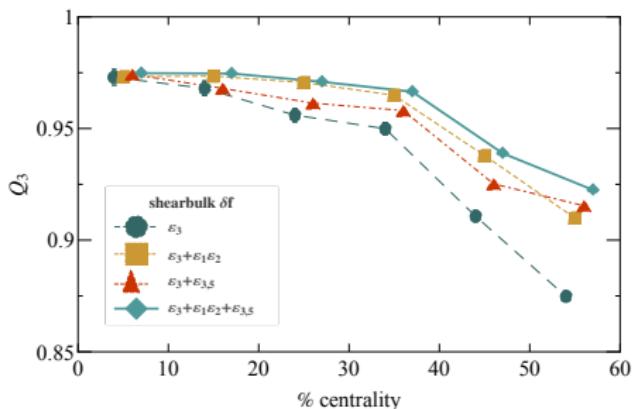
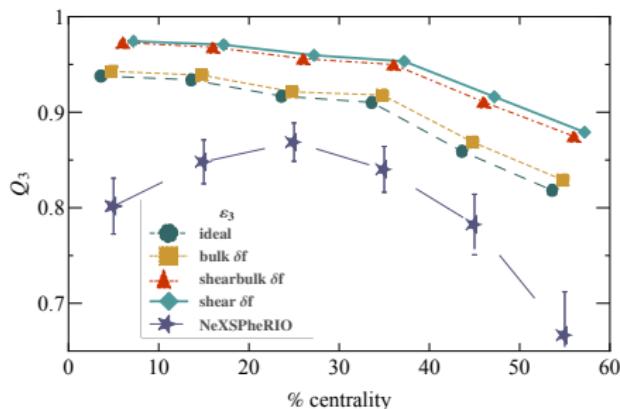
- Shear viscosity is most strongly correlated to initial conditions
- v_1 requires higher order eccentricities, correlates most strongly to low p_T for v_1



v_1 from $\varepsilon_1 + \varepsilon_2\varepsilon_3 + \varepsilon_1\varepsilon_5$

JNH arXiv:1411.2574

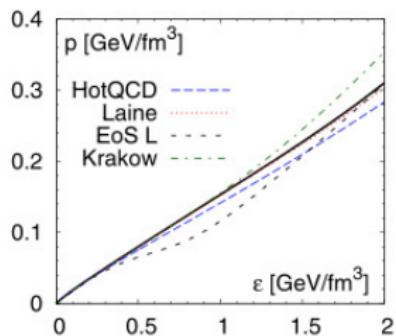
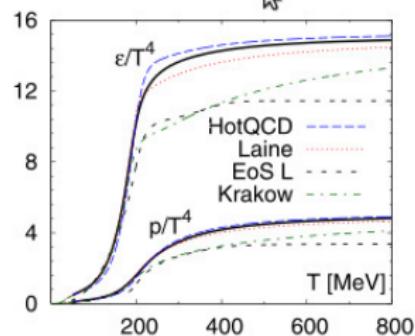
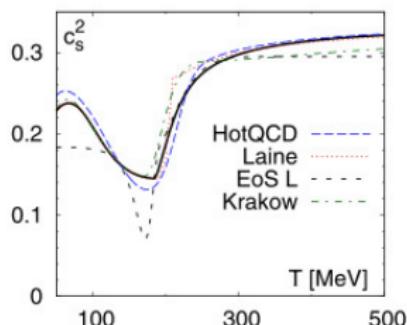
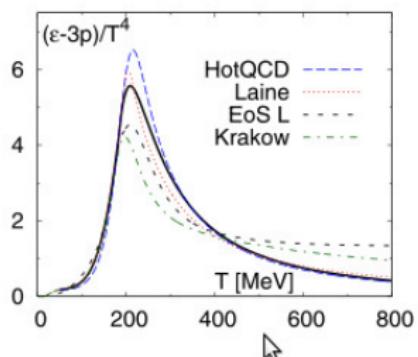
- Initial flow/3+1 dimensions decrease the correlation with the initial eccentricities (especially for central/peripheral collisions)
- Higher order eccentricities help correlate peripheral collisions



Conclusions

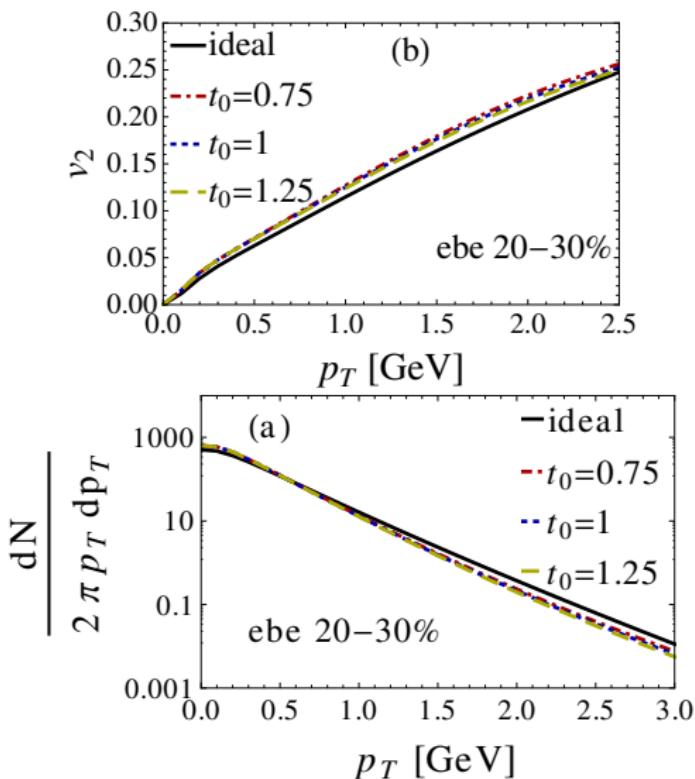
- Bulk viscosity may compensate the effects of shear viscosity (more relevant the longer the hydrodynamical calculations are done)
- When $\zeta/s = \eta/s$ the effects of bulk dominate
- Shear viscosity most strongly correlates to the initial eccentricities, shear+bulk is not as strongly correlated.
- ζ/s must be significantly smaller than η/s - otherwise runs into problems with δf .
- v-USPhydro+UrQMD results coming soon!

Equation of State



Huovinen&Petreczky, NPA837, 26(2010)

Dependence on τ_0 (bulk only)

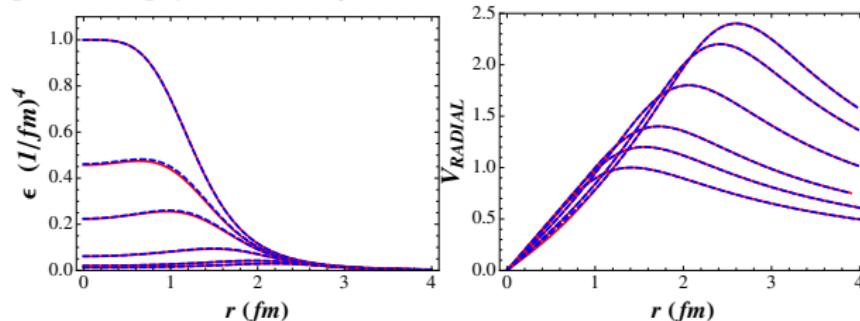


Checks- Gubser Test

- Reproduce analytical sol. from 2+1 conformal ideal hydro

$$\epsilon = \frac{\epsilon_0}{\tau^{4/3}} \frac{(2q)^{8/3}}{\left[1 + 2q^2(\tau^2 + x_\perp^2) + q^4(\tau^2 - x_\perp^2)\right]^{4/3}}$$

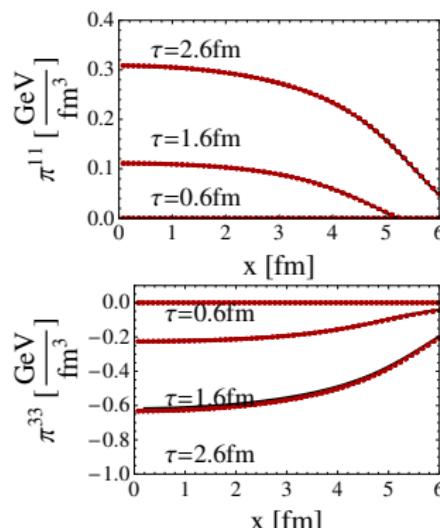
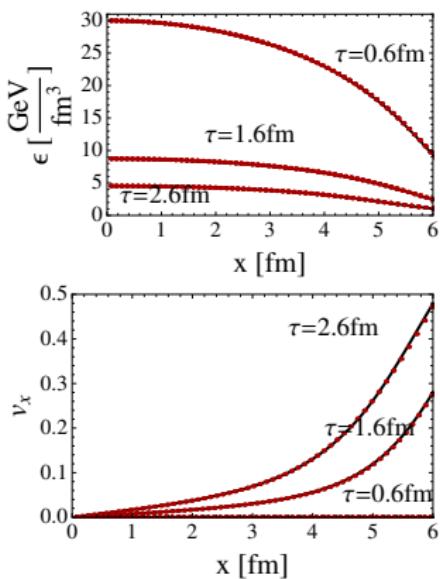
Gubser, PRD**82**, 085027(2010), Marrochio et. al. 1307.6130
[nucl-th] (first analytical solution of Israel-Stewart hydro)



- The viscous bulk evolution converges to that computed within ideal hydrodynamics for sufficiently small ζ/s .

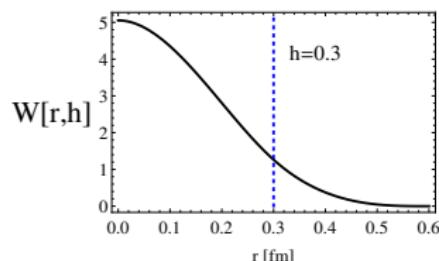
Checks- TECHQM (for shear)

Au+Au, $b = 0$ fm, EOS I ($\epsilon = 3p$), $\tau_0 = 0.6$ fm/c,



SPH Equations of Motion

- Reconstruct all hydrodynamical fields using a discrete set of Lagrangian coordinates $\{\mathbf{r}_\alpha(\tau), \alpha = 1, \dots, N_{SPH}\}$ and a normalized piece-wise distribution function $W[\mathbf{r}; h]$



- h is a length scale, determines structure
- Reference density in the lab frame

$$\tau\gamma\sigma \rightarrow \sigma^*(\mathbf{r}, \tau) = \sum_{\alpha=1}^{N_{SPH}} \nu_\alpha W[\mathbf{r} - \mathbf{r}_\alpha(\tau); h] \quad (3)$$

where ν_α are constants $\rightarrow \int d^2\mathbf{r} \sigma^*(\mathbf{r}, \tau) = \sum_{\alpha=1}^{N_{SPH}} \nu_\alpha$

SPH Equations of Motion

- Vector current becomes

$$\mathbf{j}^*(\mathbf{r}, \tau) = \sum_{\alpha=1}^{N_{SPH}} \nu_\alpha \frac{d\mathbf{r}_\alpha(\tau)}{d\tau} W[\mathbf{r} - \mathbf{r}_\alpha(\tau); h], \quad (4)$$

that satisfies $\partial_\tau \sigma^*(\mathbf{r}, \tau) + \nabla_{\mathbf{r}} \cdot \mathbf{j}^*(\mathbf{r}, \tau) = 0$

- Each "SPH particle", α , has $\mathbf{r}_\alpha(\tau)$, $\mathbf{u}_\alpha(\tau) = \gamma_\alpha(\tau) \mathbf{v}_\alpha(\tau)$, where $\mathbf{v}_\alpha(\tau) = d\mathbf{r}_\alpha(\tau)/d\tau$ and $\gamma_\alpha = 1/\sqrt{1 - \mathbf{v}_\alpha^2}$, and it carries a quantity ν_α for the reference density σ^*

SPH Variables

- For any density associated with some extensive quantity-
 $a(\mathbf{r}, \tau)$

$$a(\mathbf{r}, \tau) = \sum_{\alpha=1}^{N_{SPH}} \nu_{\alpha} \frac{a(\mathbf{r}_{\alpha}(\tau))}{\sigma^*(\mathbf{r}_{\alpha}(\tau))} W[\mathbf{r} - \mathbf{r}_{\alpha}(\tau); h] . \quad (5)$$

- Thus, entropy

$$s^*(\mathbf{r}, \tau) = \sum_{\alpha=1}^{N_{SPH}} \nu_{\alpha} \frac{s(\mathbf{r}_{\alpha}(\tau))}{\sigma(\mathbf{r}_{\alpha}(\tau))} W[\mathbf{r} - \mathbf{r}_{\alpha}(\tau); h] \quad (6)$$

the bulk term

$$\Pi(\mathbf{r}, \tau) = \sum_{\alpha=1}^{N_{SPH}} \nu_{\alpha} \frac{1}{\gamma_{\alpha} \tau} \left(\frac{\Pi}{\sigma} \right)_{\alpha} W[\mathbf{r} - \mathbf{r}_{\alpha}(\tau); h] . \quad (7)$$

SPH Variables

- Dynamical variables: $\{\mathbf{r}_\alpha, \mathbf{u}_\alpha, (\frac{s}{\sigma})_\alpha, (\frac{\Pi}{\sigma})_\alpha; \alpha = 1, \dots, N_{SPH}\}$
- Equations of Motion can then be rewritten as

$$\begin{aligned} M_\alpha^{ij} \frac{du_\alpha^j}{d\tau} &= F_\alpha u_\alpha^i + \partial^i (p_\alpha + \Pi_\alpha) \\ \gamma_\alpha (\tau_\Pi)_\alpha \frac{d}{d\tau} \left(\frac{\Pi}{\sigma} \right)_\alpha + \left(\frac{\Pi}{\sigma} \right)_\alpha &= - \left(\frac{\zeta}{\sigma} \right)_\alpha (D_\mu u^\mu)_\alpha \\ \gamma_\alpha \frac{d}{d\tau} \left(\frac{s}{\sigma} \right)_\alpha &= - \frac{1}{T_\alpha} \frac{\Pi_\alpha}{\sigma_\alpha} (D_\mu u^\mu)_\alpha \end{aligned}$$

SPH Equations of Motion

SPH discretizes the fluid into a number of SPH particles whose trajectories (\mathbf{r} and \mathbf{u}) you observe over time

Entropy

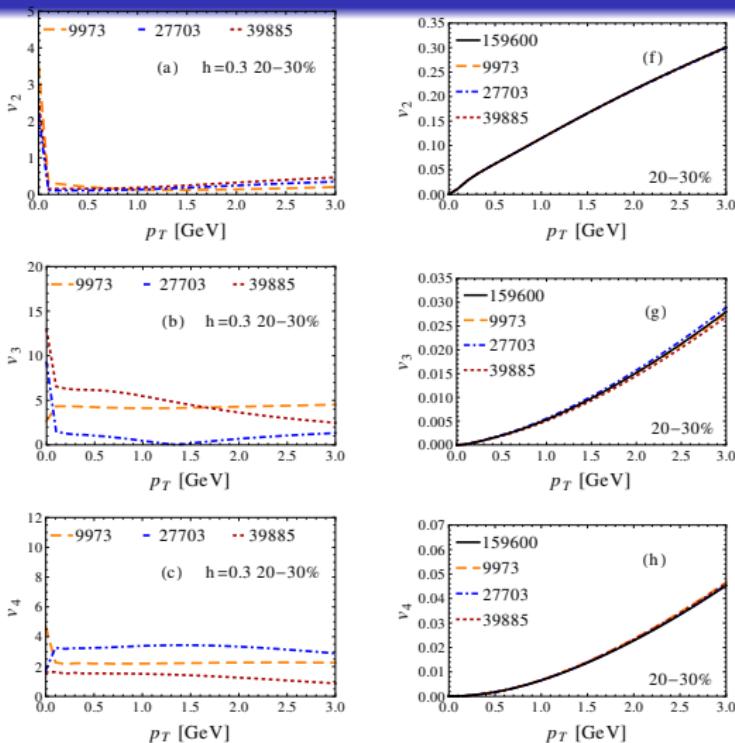
$$s^* = \sum_{\alpha=1}^{N_{SPH}} \nu_\alpha \left(\frac{s}{\sigma}\right)_\alpha W(|\mathbf{r} - \mathbf{r}_\alpha(t)|; h)$$

PDE \rightarrow ODE

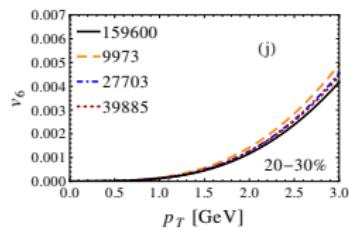
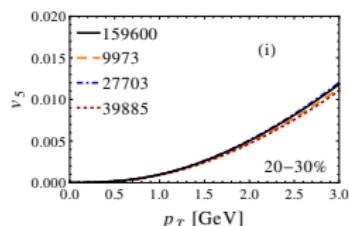
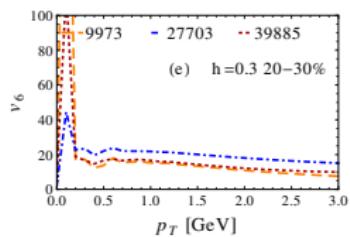
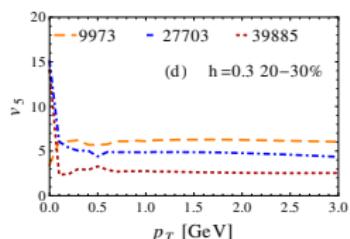
$$\begin{aligned} M_\alpha^{ij} \frac{du_\alpha^j}{d\tau} &= Btot_\alpha u_\alpha^i + F^i + \partial^i(p_\alpha + \Pi_\alpha) + v^j \partial^j \pi^{0i} - \partial^j \pi^{ij} \\ - \left(\frac{\zeta}{\sigma}\right)_\alpha (D_\mu u^\mu)_\alpha &= \gamma_\alpha(\tau_\Pi)_\alpha \frac{d}{d\tau} \left(\frac{\Pi}{\sigma}\right)_\alpha + \left(\frac{\Pi}{\sigma}\right)_\alpha \\ \gamma_\alpha \frac{d}{d\tau} \left(\frac{s}{\sigma}\right)_\alpha &= -\frac{1}{T_\alpha} \frac{\Pi_\alpha}{\sigma_\alpha} (D_\mu u^\mu)_\alpha + \frac{1}{T_\alpha} \frac{\pi_\alpha^{\mu\nu}}{\sigma_\alpha} (D_\mu u_\nu)_\alpha \end{aligned}$$

shear is much longer (not shown)

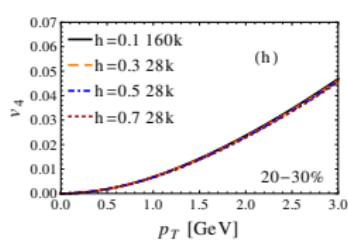
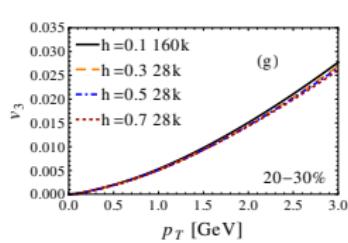
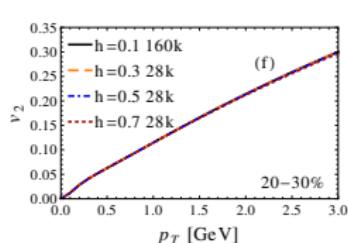
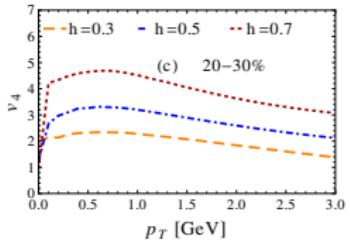
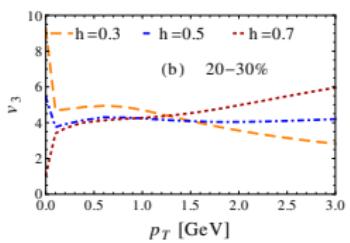
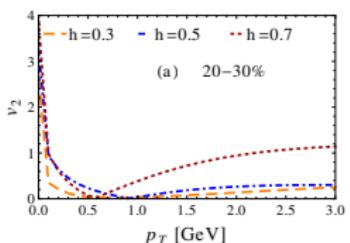
Testing of N_{SPH} with $h = 0.3$



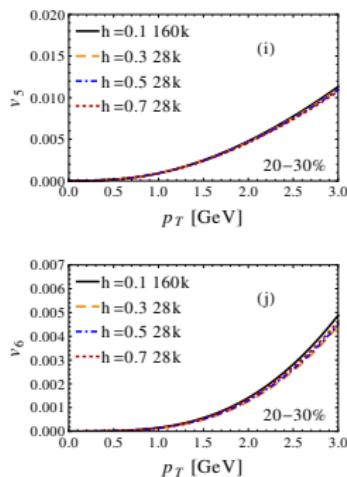
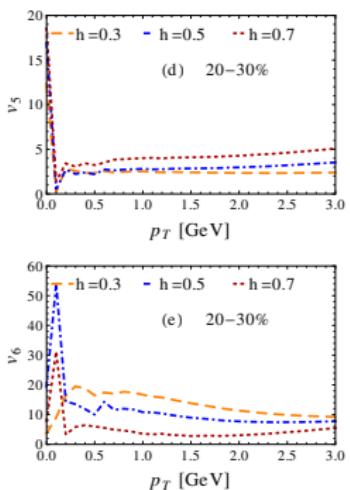
Testing of N_{SPH} with $h = 0.3$



Testing of h



Testing of h



$v_n(p_T)$'s from bulk only

